Chaotic nuclear masses?

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Outline:

Is there a chaotic component in the nuclear ground-state?

Bohigas-Giannoni-Schmit Conjecture

Periodic orbit theory

Correlations between neighboring nuclei
Is there a chaotic component in the nuclear ground-state?

Highly excited nuclei
Level statistics for neutron resonances suggests GOE
(Chaotic time-reversal invariant system)

Ground state region
Level statistics suggests Poisson
(Regular system)

O. Bohigas et. al., Nuclear Data for Science and Technology (1983)
Global mass calculations based on mean field theory, e.g. P. Möller et. al. (1995), give some error

\[ \delta E = E_{\text{exp}} - E_{\text{calc}} \]

The error seems model independent and hard to improve

\[ \delta E \sim 0.7 \text{ MeV} \]

Can \( \delta E \) be interpreted as a chaotic component in the nucleus?
Bohigas-Giannoni-Schmit Conjecture

Spectral fluctuations agree with

- **GOE** if the corresponding classical system is chaotic and obeys time-reversal symmetry.

- **Poisson** if the corresponding classical system is integrable.

Nearest neighbor distribution

![GOE and Poisson distributions](image)
Classical motion inside 2D cavities

Notice:
Level repulsion

Area filling orbits
The form factor is the Fourier transform of the two-point correlation function.

Depending on the character of the dynamics the form factor is described by Random Matrix Theory.

Chaotic GOE: \[ K(\tau) = 2\tau, \quad \tau \ll \tau_H \]

Regular: \[ K(\tau) = \tau_H \]

\[ \tau_H = \frac{h}{\delta} \]

\(\delta\) is the mean level spacing

Notice that chaotic motion does \textbf{NOT} mean random motion.

Hydrogen in a strong magnetic field is classically completely chaotic.

The quantum mechanical energy levels may be calculated with great precision.

Level statistics for hydrogen in a strong magnetic field agree well with GOE (Chaotic time-reversal invariant system).

But: Nuclear system much more complicated.
Gutzwiller's

Periodic orbit theory

Random Matrix Theory describe short energy range fluctuations of the order of the mean level distance.

Heissenberg time \( \tau_H = \hbar \rho_{\text{average}} \)

Long energy range fluctuations are described by semiclassics. This energy scale is defined by the shortest periodic orbit.

\[ E_c = \frac{\hbar}{\tau_{\text{min}}} \]

In a ballistic system this correspond to the time of flight across the system.
Mean Field theory convey a path to connect the quantum mechanical many-body problem to classical dynamics.

Reduces the fully interacting many-body problem to single-particle motion in a self-consistent potential.

The basic object is the single particle level density

$$\rho(E,x) = \sum_i \delta(E - E_i(x))$$

In a semiclassical $\hbar$-expansion

$$\rho(E,x) = \rho_{\text{average}}(E,x) + \rho_{\text{fluct}}(E,x)$$
The trace formula

Leading order in an $\hbar$-expansion gives a sum over classical periodic orbits

$$\rho_{\text{fluct}}(E, x) = 2 \sum_{p, r} A_{p, r}(E, x) \cos \left( \frac{rS_p(E, x)}{\hbar} + \nu_{p, r} \right)$$

- **Stability amplitude**
- **Classical action**
- **Maslov index**

$$S_p = \oint p \, dq$$

$$H_{QM}(x) \leftrightarrow H_{CM}(q, p)$$
In a system of non-interacting fermions the fluctuating part of the total energy may be written

\[ E_{\text{shell}}(x) = \int_0^\mu \rho_{\text{fluct}}(E, x) E \, dE \]

\[ E_{\text{shell}}(x) = 2 \hbar^2 \sum_{p, r} A_{p, r}(x) \frac{A_{p, r}(x)}{r^2 \tau_p^2} \cos \left( \frac{rS_p(x)}{\hbar} + \nu_{p, r} \right) \]

The orbits are evaluated at the Fermi energy \( \mu \)

\[ \int_0^\mu \rho_{\text{average}}(E, x) \, dE = A \]

The second moment may be written as

\[ \langle E_{\text{shell}}^2 \rangle \approx \frac{\hbar^2}{2\pi^2} \int_0^\infty \frac{d\tau}{\tau^4} K(\tau) \]

where \( K(\tau) \) is the form factor.

Typical size of energy contributions:

\[ \sigma_{\text{chaotic}} = \sqrt{\langle E_{\text{shell}}^{\text{chaotic}}^2 \rangle} = \frac{2.8}{A^{1/3}} \text{MeV} \]

\[ \sigma_{\text{reg}} = \sqrt{\langle E_{\text{shell}}^{\text{reg}}^2 \rangle} = 2.8 \text{MeV} \]

Independent of any detailed information concerning the many-body problem.

Only parameter is \( \tau_{\text{min}} \), which is related to the size of the nucleus.

Provides an order of magnitude estimate and fixes an energy scale where a new phenomenon appears.
Comparison between $\sigma_{ch}$ and the typical size of the error $\delta E$ of the P. Möller. et. al. (1995) mass formula.
Comparison between $\sigma_{ch}$ and the typical size of the error $\delta E$ of different mass formulas.

Correlations between neighboring nuclei

When the external parameter $x$ is varied, the autocorrelation function is defined as

$$C(x) = \langle E_{shell}(x_0 - x/2) E_{shell}(x_0 + x/2) \rangle_{x_0}$$

Leads to a double sum with interferent terms between different periodic orbits.

The main contribution comes from the shortest orbits, i.e. those with shortest period.
Diagonal approximation

\[ C(x) = 2\hbar^4 \sum_{p,r} \frac{A^2_{p,r}}{r^4 \tau^4_p} \cos \left( \frac{rQ_p}{\hbar} x \right) \]

\[ Q_p = \frac{\partial S_p}{\partial x} \]

Using the spectral form factor \( K(\tau) \)

\[ C(x) = \frac{\hbar^2}{2\pi^2} \int_0^\infty \frac{d\tau}{\tau^4} \left\langle \cos \left( \frac{Q_p x}{\hbar} \right) \right\rangle_\tau K(\tau) \]

So far valid for regular or chaotic motion

The kind of dynamics (chaotic or regular) determine \( Q_p \) and \( K(\tau) \)
Final result for the normalized correlation function assuming chaotic dynamics

\[ C_N(\zeta) = (1 - \frac{\zeta^2}{4}) e^{-\frac{\zeta^2}{4}} + \frac{\zeta^4}{16} \Gamma(0, \frac{\zeta^2}{4}) \]

\[ \Gamma(a, z) = \int_z^\infty t^{a-1} e^{-t} \, dt \]

Universal function since all the system dependent features are hidden in the unfolding

\[ \zeta = \sqrt{\frac{2 \alpha \tau_{\text{min}}}{\hbar}} x \]

\[ \zeta = \sqrt{\frac{\langle (\partial_x E_{\text{shell}}(x))^2 \rangle}{\langle E_{\text{shell}}(x)^2 \rangle}} x \]

\( \alpha \) is a system dependent parameter which is not accessible experimentally, express unfolding in properties directly related to the nucleus
Experimental correlations between neighboring nuclei

Define the normalized correlation function

\[ C(dN) = \frac{\langle \delta E(Z, N) \delta E(Z, N + dN) \rangle_N}{\langle \delta E^2 \rangle_N} \]

Where \( \delta E \) is the difference between calculated and measured masses.

Define rescaling in order to compare to theory:

\[ \zeta = \sqrt{\frac{\langle (\delta_{Z_{\text{in}}} \delta E)^2 \rangle}{\langle \delta E^2 \rangle_N}} dN \]
Experimental correlation functions compared to periodic orbit theory assuming chaotic motion

Möller
Experimental correlation functions compared to periodic orbit theory assuming chaotic motion.
Conclusions

• Periodic orbit theory describe the regular and the chaotic part of the shell energy on equal footing.

• The second moment of the chaotic shell energy agree well with the typical size of the error in nuclear mass formula.

• The correlations for the error between neighboring nuclei agree well with estimates from periodic orbit theory assuming chaotic dynamics. The result is quite different from uncorrelated white noise.

• Periodic orbit theory predicts definite non-random fluctuations. It may be a difficult task to compute the chaotic contribution for each nucleus, but periodic orbit theory puts NO a priori physical barrier to the accuracy of the theoretical mass calculations.
These conclusions give further support to the conjecture that the error in the computation of the nuclear masses is related to chaotic motion.