Lunds Universitet — Spring 2016

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Exercises to Quantum Mechanics FYSN17/FMFN01, Week 4

Homework to be handed in until February, 16 in Johannes' mailbox

Exercise 1: *Sum of angular momenta*

Aim: Realizing the meaning of coupling between angular momenta

Show that the sum of two angular momenta satisfies the basic commutation relations for angular momentum. The two angular momenta that are summed operate in different spaces. (Examples: $\hat{\mathbf{S}} = \hat{\mathbf{S}}_1 + \hat{\mathbf{S}}_2$ and $\hat{\mathbf{J}} = \hat{\mathbf{L}} + \hat{\mathbf{S}}$.)

Exercise 2: *Positronium state*

Aim: Getting familiar with typical eigenstates of total angular momentum

Show that the state $|\Psi\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle_e \otimes |\downarrow\rangle_p + \frac{1}{\sqrt{2}}|\downarrow\rangle_e \otimes |\uparrow\rangle_p$ is an eigenstate of $\hat{S}_z$ and $\hat{\mathbf{S}}^2$, where $\hat{\mathbf{S}} = \hat{\mathbf{S}}_e + \hat{\mathbf{S}}_p$. (notation as in the lecture notes)

Exercise 3 (Homework): *Clebsch-Gordan coefficients*

Aim: Evaluating Clebsch-Gordan coefficients for a specific scenario

The state of an electron is

$$|\Psi\rangle = a|l=2, m=0\rangle \otimes |\uparrow\rangle + b|l=2, m=1\rangle \otimes |\downarrow\rangle\,,$$

where a and b are constants with $|a|^2 + |b|^2 = 1$.

a) Without making any assumptions about the total angular momentum of this state, calculate a and b such that $|\Psi\rangle$ is an eigenstate of the following operators: $\hat{\mathbf{L}}^2$, $\hat{\mathbf{S}}^2$, $\hat{\mathbf{J}}^2$, and $\hat{J}_z$.

b) Explain what you found in a) by analysing $|\Psi\rangle$.

Exercise 4: *LS-coupling*

Aim: Understanding the difference between both basis states; Training to determine eigenstates

An electron is in a shell with $l = 1$. Determine the matrix for the perturbation $\hat{\mathbf{H}}_{LS} = k\hat{\mathbf{L}} \cdot \hat{\mathbf{S}}$ in the following cases:

a) In the basis $\{|j = 3/2, \{m_j\}, l = 1, s = 1/2\rangle, |j = 1/2, \{m_j\}, l = 1, s = 1/2\rangle\}$, where m_j runs over all allowed indices.

b) In the basis $\{|l = 1, m\rangle \otimes |\uparrow\rangle, |l = 1, m\rangle \otimes |\downarrow\rangle\}$, where m runs over all allowed indices.

Exercise 5 (Homework): *Effective g-factor*

Aim: Working with different basis states of coupled angular momentum; Understanding an important concept of atomic physics

Determine the matrix for

$$\hat{\mathbf{H}}_p = \hat{\mathbf{H}}_{LS} + \hat{\mathbf{H}}_{\text{external}} = k\hat{\mathbf{L}} \cdot \hat{\mathbf{S}} + \frac{eB}{2m}(\hat{L}_z + 2\hat{S}_z)$$

in the basis of exercise 4. ($g_e = 2$ is used here). While the multiplets are degenerate for vanishing magnetic field, they split up with increasing B and couple with each other. Evaluate the magnitude of the splitting, if you neglect the off-diagonal elements between the $j = 3/2$ and $j = 1/2$ levels. (In the spririt of perturbation theory this is allowed provided the LS coupling is much larger than the Zeeman energy.) Determine the effective $g-$factors, g_{eff}, such that $E(m_j) = E_0 + g_{\text{eff}}\mu_B B m_j$.

Hint: Write the kets of the total angular momentum basis in terms of the local spin basis using the Clebsch-Gordan tables.

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Exercise 6: *Symmetry of 2-spin state*

Aim: Considering Spin-Spin coupling from a many-body perspective

Consider the coupling of two spins with $s = 1/2$. Provide the common eigenstates of $\hat{\mathbf{J}}^2$ and $\hat{J}_z$, where $\hat{\mathbf{J}} = \hat{\mathbf{S}}^{(1)} + \hat{\mathbf{S}}^{(2)}$ is the operator of the total spin. Are these states eigenstates of the permutation operator? Are they symmetric or anti-symmetric?